How quorum rules distort referendum outcomes: evidence from a pivotal voter model

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Abstract

In many jurisdictions, whether referendum results are binding depends on legally defined quorum requirements. We use a pivotal voter model to examine the consequences of such requirements. We find that, although quorum rules differ in consequences, a status quo bias that is usually attributed need not be present and that quorum rules may work against the status-quo. The rules can also both favor minorities and reduce voter turnout. Because quorum rules can create situations in which the secrecy of the vote is compromised, the door is opened to undemocratic forms of social and political pressure.

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1. Introduction

How do quorum rules affect the extent to which referendum results correctly represent collective preferences on a particular issue? Quorum requirements are part of the 'bewildering variety' of institutional rules that govern direct democracy (Matsusaka, 2005). In many jurisdictions, whether a voting result is accepted as valid is contingent on turnout reaching a legally defined threshold (participation/turnout quorum), or on a majority of supporters of a given change to the status quo representing a particular share of the electorate (approval quorum). What are the consequences of such quorum requirements for referendum outcomes compared with referenda determined by a simple majority rule?

The answer is of considerable scholarly and practical importance. Direct democracy is becoming increasingly popular in the United States and Europe and also in Latin American and East Asia. The number of jurisdictions with direct democracy mechanisms has increased in the last decades, as has the frequency with which votes on referenda and initiatives take place.[1] It has become impossible to discuss developments in several policy areas – such as abortion rights, gay rights, European integration, constitutional reforms or local budgetary politics – without reference to role played by direct democracy.[2]

Moreover, referenda are receiving increasing support not only from citizens in established democracies,[3] but also from the scholarly community itself, which has begun increasingly to stress advantages of direct democracy rather than its shortcomings (Matsusaka and McCarty, 2001).[4] Traditional concerns with decision-making costs involved in direct democracy have been balanced by findings suggesting that policy decisions might be brought closer to median-voter preferences (Matsusaka, 2004), foster policy compromises (Miguet, 2008), break the control of party cartels over the political agenda (Gerber, 1996), raise levels of information about issues (Smith and

[1] See Smith (1986), Butler and Ranney (1994), Setälä (1999), Barczak (2001), Le Duc (2003), Qvortrup (2005), and Matsusaka (2005).

[2] See, among many, Roh and Haider-Markel (2003), Hug and Sciarini (2000), Christin and Hug (2002), Hug (2003), Clarke and Kornberg (1994), Pattie et al. (1999), Megdal (1993), and Feld and Matsusaka (2003).

[3] See Dalton et al. (2001), Donovan and Karp (2006), and Bowler et al. (2007).

[4] Frey (1994) argued that direct participation is a way to keep agenda-setting power with the voters and that fears of 'irresponsible voters' or of the 'excesses of majority' do not have a real basis as long as the institutional framework is adequate.

Tolbert, 2004), and positively affect both objective (Blomberg et al., 2004) and subjective well-being (Frey and Stutzer, 2000).[5]

Arrow's impossibility theorem (1950) applies to situations where choices are made over three or more discrete options. Referenda with binary decisions may therefore be a way to avoid Arrow's impossibility theorem. However, the interesting properties that referenda may have are likely to hinge on the specifics of institutional design. As May (1952, p. 683) wrote, 'any social welfare function (…) that is not based on simple majority rule, i.e., does not decide between any pair of alternatives by majority vote, will either fail to give a definite result in some situation, favor one individual over another, favor one alternative over the other, or fail to respond positively to individual preferences.' Dasgupta and Maskin (2008), using a different characterization, also showed that simple majority rule satisfies some desirable axioms over a larger class of preference domains than any other voting rule. Thus, voting schemes that deviate from simple majority rule in binary decisions are bound to fail to represent individual preferences, and can cause voting paradoxes or provide incentives for voters to hide their true preferences.

Quorum rules in referenda introduce such consequences. The rationale behind their adoption is to prevent active minorities from imposing their will upon a passive majority. The rules are supposed to 'prevent low turnout from producing a distorted result' (Le Duc, 2003, p. 172) and to be an 'effective safeguard against so called "false" majorities (a minority's exploitation of voter apathy)' (Qvortrup, 2005, p. 173).

No less than fourteen of the European Union's member-states have some combination of turnout or approval quorum rules with regard to referenda (see Aguiar-Conraria and Magalhães, forthcoming). In the majority of states in the German Federal Republic, state and municipal referenda are valid only if an approval quorum is reached (Kaufmann et al., 2008). Several American states, such as Wyoming and Minnesota (approval quorum) and Massachusetts, Mississippi, and Nebraska (turnout quorum) have adopted such rules, which can also be found in Colombia, Belarus, Venezuela and Taiwan (Qvortrup, 2005; IDEA, 2008; Herrera and Matozzi, 2010).

[5] See also Bowler and Donovan (1998), Hajnal et al. (2002), Benz and Stutzer (2004), and Torgler (2005), among others. For a review of the literature, see Lupia and Matsusaka (2004).

Despite this, and in contrast with other aspects of the institutional design of referenda,[6] there has been a lack of systematic rigorous examination of the rules. Among the few exceptions are Côrte-Real and Pereira (2004), who use a decision-axiomatic approach to show that different quorum types imply different assumptions about the interpretation of abstention. Herrera and Mattozzi (2010), in a group turnout model, show that both turnout and approval quorum requirements affect the behavior of politicians. In their model, the incentive of each voter to vote depends solely on how much money each party spends on campaigning. The existence of quorum requirements allows parties that favor preserving the status quo to use a 'quorum-busting strategy'.[7] However, because this strategy may be unsuccessful, it is possible that the demobilization of the supporters of the status quo results in increased probability that the proposal for change wins. They also show that, in the context of their model, participation and approval requirements are equivalent. Finally, Aguiar-Conraria and Magalhães (forthcoming), in an empirical examination of the subject using data on all national referenda held in the current European Union countries from 1970 to 2006 find significant negative effects of participation quorum prerequisites on turnout levels. Their results contrast with those of Herrera and Mattozzi (2010) by finding that approval quorum requirements have, if anything, a positive (rather than negative) impact on turnout rates.[8]

Our purpose in this paper is to examine these and other aspects of the consequences of quorum rules. More specifically, we are interested in examining whether quorum rules, by introducing deviations from simple majority rule in referenda, can have consequences for referendum outcomes (including turnout), and whether those outcomes are likely to correctly reveal collective preferences.

Herrera and Mattozzi (2010) have addressed the issue of turnout under quorum requirements using a group-based model of turnout, where effort exerted by parties based on imperfect information about voters' preferences determines whether supporters vote. However, although this approach focuses on a potentially important

[6] See, for example, Hug and Tsebelis (2002), Lupia and Matsusaka (2004), Hug (2004), Freitag and Vatter (2006).

[7] For generic non-formal presentations of the same argument regarding turnout quorum rules, see Venice Commission (2007) and, regarding the Italian case, Uleri (2002).

[8] On a related issue, Felsenthal (1991) examined the conditions under which decision-making bodies that are not fully assembled will reach the same decision as if they were, and concludes that, if a special majority is considered necessary, that special majority should correspond to a percentage of entire electorate (i.e., analogous to an approval quorum). Other related work is Zwart (2009), who argues that optimal quorum rules depend on the propensity to vote of different interest groups.

aspect of any election — mobilization — it inevitably sees voters as passive and neglects the micro-level strategic decision making process on their part.

We examine the issues of the effects of quorum requirements using a different framework, that of the pivotal-voter model originally proposed by Ledyard (1984) and Palfrey and Rosenthal (1985). Like Herrera and Mattozzi, we also find that quorum requirements may produce several paradoxical and detrimental consequences, and some of our concrete findings are in line with theirs, showing their robustness to the choice of environment. More specifically, we also conclude that quorum requirements may reduce turnout; that they do not always introduce a bias for the status quo; and that, unlike the rationale often invoked for their introduction, they can favor minorities imposing their will on majorities.

Our findings contrast with Herrera and Mattozzi in other ways. First, we do not conclude that a participation quorum is equivalent to an approval quorum. On the contrary, we conclude that the channels by which incentives change are different. Both quorum requirements promote abstention among 'status quo' supporters. However, while participation quorum rules give the latter strong reasons to abstain, approval quorum rules leave them without any reason to vote. In other words, as we shall explain later in more detail, while a participation quorum promotes strategic abstention, an approval quorum promotes passive abstention. Second, for a given choice of parameters, we do not obtain unique equilibria. Unlike in Herrera and Mattozzi's setup, we find that it is not possible to predict a single outcome and to backtrack voters' preferences even if the parameters are known, something that, in fact, should make policymakers even wearier of adopting quorum rules. Finally, we show that quorum rules create situations under which one of the basic principles of democracy, the secrecy of the vote, becomes seriously imperiled, opening the door to undemocratic forms of social and political pressure. This is so because, rather than producing unique equilibria with strictly positive participation rates, in our model, the existence of a quorum requirement always implies the existence of 'no show' equilibria.

The paper proceeds as follows. In section 2, we introduce the pivotal voter model as applied to referenda with a binary choice, and modify it to accommodate simple majority voting, turnout, and approval quorum rules. In sections 3 to 5, we show, by computational simulations for different parameter values - capturing the preferences among the electorate and the intensity of such preferences - that several additional equilibria emerge because of quorum rules. In section 6, we discuss the robustness of our results to the introduction of consumption or expressive voters, as opposed to pure instrumental voters. Section 7 concludes.

2. A pivotal voter model

In a pivotal voter model, citizens rationally anticipate the probability that their votes will be pivotal and they will vote if the expected benefit outweighs the cost of voting. Without quorum requirements, a strictly positive level of turnout is assured in equilibrium: if no citizens were expected to vote, any deviator would be pivotal with probability one. The pivotal voter model is widely used and yields several implications. For example, Campbell (1999) shows that the well-known phenomenon of small minorities with strong feelings imposing their views on an apathetic majority is well accommodated by the Palfrey and Rosenthal model, while Börgers (2004) shows that voluntary voting Pareto-dominates compulsory voting. Although it falls short of solving the paradox of voting for large elections, the pivotal vote model remains widely used, since many of its implications and comparative static predictions are supported by empirical evidence (Blais, 2000; Levine and Palfrey, 2007; Palfrey, 2009). We take our support model from Coate et al. (2008), which is a traditional pivotal voter model, and modify it to accommodate different quorum rules. We focus on referenda with a binary choice:

1. The 'Yes' option: change the 'status quo';
2. The 'No' option: preserve the 'status quo'.

Those who support the 'Yes' option are described as 'changers' or 'supporters'. Those who support the 'No' option are the 'conservatives' or 'opponents'.

Each citizen must decide whether or not to vote. It is trivial to show that, if they choose to vote, they will vote according to their preferences: changers will vote 'Yes', conservatives will vote 'No'.

We assume that there are n registered voters $(i = 1, \dots, n)$ and that each faces a cost of voting given by c_i, where c_i is the realization of a uniformly distributed random variable, $c_i \sim U[0, c]$. Like Campbell (1999), Börgers (2004), and Coate et al. (2008), we rule out negative costs. A voter with a negative cost would never abstain. Given that our objective is to study how the existence of a quorum rule may change the incentives to vote, it seems reasonable that we rule out voters that would vote (or abstain) no matter what. In any case, in section 6, we will examine the consequences of relaxing the assumption of non-negative costs.

If ‘Yes’ wins the election, then supporters obtain a benefit b, while opponents incur a loss x. Each voter knows his or her own type and knows the probability, μ, that each individual voter favors the proposal. Each voter has perfect knowledge about own costs but only knows the distribution of the costs of the other voters.

A voter derives utility from voting only if pivotal, i.e. if he or she casts the decisive vote. The probability of being pivotal depends on the strategies of the other voters. This is a game of incomplete information, with preferences and costs exogenously given. The ‘Yes’ option wins only if it receives more votes than the ‘No’ option and the quorum requirements are satisfied.

A strategy for voter i is a function that specifies if he or she votes or abstains for each possible realization of c_i. We look for symmetric Bayesian-Nash equilibria: given the strategies of the other citizens and the distribution of supporters and voting costs, each citizen must be happy with his or her own strategy. By symmetry, we mean that all members of a group (changers or conservatives) follow the same strategy. A voter will vote if the voting cost is below some threshold. Let γ_s and γ_o be those cut-off values for supporters and opponents, respectively.

Without quorum requirements, and given our tiebreak rule (‘No’ wins if there is a tie), our model is exactly the same model of Coate et al. (2008). Alternatively, we could have assumed, like Börgers (2004), that if both alternatives received exactly the same number of votes, each would be selected with probability 1/2. In our case, for computational reasons, we have not chosen this latter option.[9] However, Palfrey and Rosenthal (1985) argue that their main theorems are robust to several tiebreaking rules, which include these two. Apart from a small asymmetry between changers and conservatives introduced in the model, this assumption has negligible effects on the results and simplifies computations by a great deal.

[9] Note that if the tiebreak is decided by tossing a coin, then a voter can be pivotal in two different ways: (1) when her vote is decisive to guarantee a tie and (2) when her vote is decisive to break the tie.

2.1. The model with no quorum requirements

We start by assuming no frictions, meaning no quorum requirements: the 'Yes' option wins if and only if it receives more votes than the 'No' option.

Consider the choices that voter i faces. Assuming that the remaining $n-1$ registered voters follow their equilibrium strategies, each changer will vote if his or her voting cost is less than γ_s, while conservatives will vote if their voting cost is less than γ_o. Let $\rho(v_s, v_o; \gamma_s, \gamma_o, c)$ be the probability that, among the $n-1$ individuals, v_s vote 'Yes' and v_o vote 'No', given their equilibrium strategies, γ_s and γ_o, and the voting cost distribution.

Given our tiebreak rule, a changer will be pivotal when, among the $n-1$ other registered voters, the number of 'Yes' votes is equal to the number of 'No' votes. In this case, the changer's vote is decisive to guarantee victory for the 'Yes' option.

Therefore, the expected benefit of voting is $\sum_{v=0}^{\frac{n}{2}-1}[\rho(v, v; \gamma_s, \gamma_o, c)]b$.

He or she will vote if the expected benefit exceeds the cost of voting. In equilibrium, this means that

$$\sum_{v=0}^{\frac{n}{2}-1}[\rho(v, v; \gamma_s, \gamma_o, c)]b = \gamma_s \tag{1}$$

For the opponents, the reasoning is analogous. A conservative will be pivotal when, among the $n-1$ other registered voters, the number of 'No' votes is equal to the number of 'Yes' votes minus one. The expected benefit of voting is $\sum_{v=0}^{\frac{n}{2}-1}[\rho(v+1, v; \gamma_s, \gamma_o, c)]x$.

In equilibrium, we have

$$\sum_{v=0}^{\frac{n}{2}-1}[\rho(v+1, v; \gamma_s, \gamma_o, c)]x = \gamma_o. \tag{2}$$

We have two equations (1 and 2) and two unknowns. To compute the equilibrium we need to derive the function $\rho(v_s, v_o; \gamma_s, \gamma_o, c)$.

As we have defined before, μ is the probability that each individual is a 'Yes' supporter. Therefore, the probability, $P(s)$, that there are s supporters among the remaining $n-1$ registered voters is given by $P(s) = \binom{n-1}{s} \mu^s (1-\mu)^{n-1-s}$.

Among the s supporters, only the ones whose individual costs are smaller than the expected benefits will vote. Therefore, the probability that v_s of those will vote is $V(v_s) = \binom{s}{v_s} \left(\frac{\gamma_s}{c}\right)^{v_s} \left(1-\frac{\gamma_s}{c}\right)^{s-v_s}$.

Similarly, the probability that, among the other $n-1-s$ registered voters, v_o will vote 'No' is $V(v_o) = \binom{n-1-s}{v_o} \left(\frac{\gamma_o}{c}\right)^{v_o} \left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v_o}$.

Putting all these equations together, we have $(v_s, v_o; \gamma_s, \gamma_o, c) =$ $= \sum_{s=v_s}^{n-1-v_o} \binom{s}{v_s} \left(\frac{\gamma_s}{c}\right)^{v_s} \left(1-\frac{\gamma_s}{c}\right)^{s-v_s} \binom{n-1-s}{v_o} \left(\frac{\gamma_o}{c}\right)^{v_o} \left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v_o} P(s)$.

Introducing this expression in equations 1 and 2, we have a system of two nonlinear equations and two unknowns.

Existence of a solution is not a problem (see Ledyard, 1984; Palfrey and Rosenthal, 1985; or Coate et al., 2008), but there are no general uniqueness results. Multiple solutions are possible. To our knowledge, the only uniqueness result derived so far is for the case of $\mu = 0.5$ and $b = x$ (Börgers, 2004).

2.2. Graphical illustration

Before changing the model to incorporate quorum requirements, it may be useful to introduce the main ideas with the aid of a simple picture. In Figure 1, let the vertical axis represent the percentage of the population that favors the proposal submitted to referendum. In the horizontal axis, we have the percentage of people that oppose the proposal. If there are no quorum requirements (Figure 1.a), there is a change in the status quo, if the outcome of the referendum places the results above the 45-degree line (meaning that the majority of the voters vote 'Yes'). Therefore, a supporter who believes that the referendum outcome will be placed on the thick black line believes that he or she will be pivotal. On the other hand, a conservative believes that he or she will be pivotal if he or she believes that the outcome will be on the gray line.

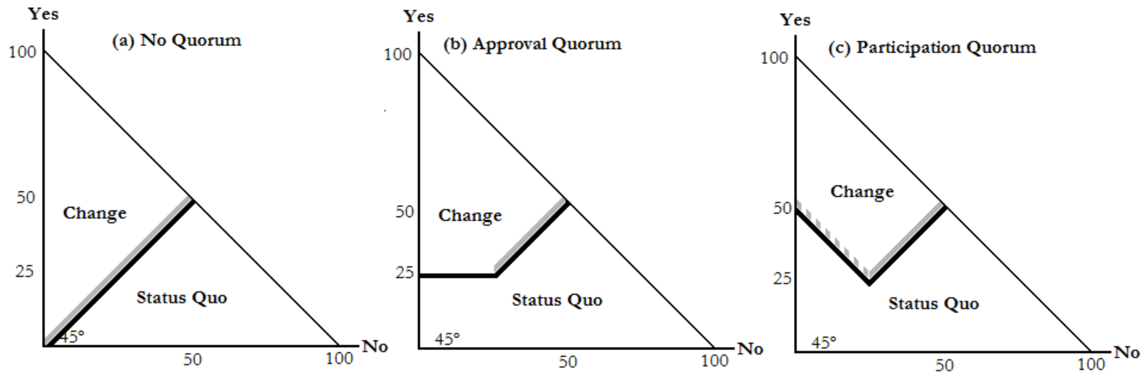

Figure 1: Pivotal lines

Figure 1.b describes a situation where there is an approval quorum of 25%. Here, a change in the status quo requires the results to be above the 45-degree line and on or above the 25%-Yes line. In this case, the ‘Status Quo’ region increases. There is also a qualitative change on the probability of being pivotal. A supporter may be pivotal if his or her vote is decisive to surpass 50% of the votes (given that the quorum is satisfied) or to reach the quorum (given that the majority votes ‘Yes’). The thick black line represents these possibilities. For the opponent, the situation is different. A ‘No’ vote that guarantees a ‘No’ majority is decisive only if the quorum is met. Therefore, the pivotal gray line is reduced, when compared to the first case.

Finally, Figure 1.c describes a situation where there is a participation quorum of 50%. The ‘Change’ region is reduced to the area on or above the 50% participation rate and above the 45-degree line. A supporter is pivotal if his or her vote is necessary to surpass 50% of the votes (given that the quorum is met) or to meet the quorum (given that the majority is guaranteed). The thick black line represents this. A conservative may have the decisive vote in a favorable way (concurrent with his or her preferences) and in pernicious way (conflicting with his or her preferences). Given that the quorum is satisfied, a conservative is pivotal, in a favorable way, if his or her vote guarantees that ‘No’ reaches at least 50% of the votes (thick gray line). He or she is pivotal in a pernicious way when his or her vote is decisive to meet the quorum requirement in a situation where ‘Yes’ has a majority (dashed gray line).

From the pictures, the main objective of imposing a quorum requirement becomes clear. The idea is to create a bias for the status quo, by enlarging the ‘Status Quo’ region. This way, it would be more difficult for very active minorities to change the status quo. Of course, if the minority represents less than 25% of the electorate, this objective is achieved. But what if the minority represents, say, 30% of the electorate? We will see in the following sections whether this objective is actually achieved.

2.3. The model with an approval quorum requirement

If there is an approval quorum, a change in the 'status quo' requires two conditions: (1) the majority of votes and (2) that that majority represents a certain percentage of the total electorate. In our calculations, we will consider that certain percentage to be 25%.

For a conservative, the only modification one has to make to equation 2 is to consider the summation from $v = n/4 - 1$ instead of $v = 0$:

$$\sum_{v=\frac{n}{4}-1}^{\frac{n}{2}-1} \left[\rho_{aq}(v+1, v; \gamma_s, \gamma_o, c)\right] x = \gamma_o \tag{3}$$

This modification happens because if the number of 'Yes' votes is smaller than $v = n/4$ the 'status quo' wins, independently of who receives the majority of the votes.

For a person who favors the proposal, there are two possibilities of being pivotal. If the quorum is satisfied, a changer is pivotal if, among the other voters, v vote 'Yes' and v vote 'No'. On the other hand, the voter can also be pivotal if his or her vote is decisive to guarantee that the quorum is satisfied. In equilibrium, we have:

$$\sum_{v=\frac{n}{4}}^{\frac{n}{2}-1} \left[\rho_{aq}(v, v; \gamma_s, \gamma_o, c)\right] b + \sum_{v=0}^{n/4-1} \rho_{aq}\left(\frac{n}{4} - 1, v; \gamma_s, \gamma_o, c\right) b = \gamma_s. \tag{4}$$

The system of equations to solve is given by equations (3) and (4).

Again, existence of a solution is not a problem, as not voting is always an equilibrium strategy. To realize this, just note that if one believes that nobody else is going to vote, his or her incentives to vote are zero. Whether he or she is a supporter (the vote is not enough to meet the quorum), or an opponent (the 'status quo' will win any way) is irrelevant. Uniqueness is not guaranteed either, as it will be abundantly illustrated in section 3.

2.4. The model with a participation quorum requirement

With a participation quorum, a change in the status quo requires two conditions: (1) the majority of the votes and (2) that the turnout rate is above some threshold. We will consider that turnout rate, the percentage of registered voters that take part in the vote, to be 50%.

For a person that supports the proposal, the modifications to introduce to equation 1 are straightforward:

$$\sum_{v=\frac{n}{4}}^{\frac{n}{2}-1} \rho_{pq}(v, v; \gamma_s, \gamma_o, c)b + \sum_{v=\frac{n}{4}+1}^{\frac{n}{2}} \rho_{pq}\left(v-1, \frac{n}{2}-v; \gamma_s, \gamma_o, c\right) b = \gamma_s. \tag{5}$$

The first term applies when the quorum is met, meaning that an voter is pivotal if, among the other voters, v vote 'Yes' and v vote 'No'. The second term captures the possibility of being pivotal when his or her vote is decisive to guarantee that the quorum is satisfied — note that $(v-1)+(n/2-v)=(n/2-1)$.

The most interesting case is for a person who opposes a proposal. A conservative can be pivotal in two conflicting ways. He or she can be pivotal because his or her vote is decisive to guarantee that the 'No' votes tie with the 'Yes' votes. However, he or she can also be decisive to meet the quorum requirement. In such case, even if the person votes 'No', his or her vote is decisive to guarantee that the 'Yes' wins. Therefore, his or her utility decreases. Accordingly, we have:

$$\sum_{v=\frac{n}{4}}^{\frac{n}{2}-1} \rho_{pq}(v+1, v; \gamma_s, \gamma_o, c)x - \sum_{v=\frac{n}{4}+1}^{\frac{n}{2}-1} \rho_{pq}\left(v, \frac{n}{2}-v-1; \gamma_s, \gamma_o, c\right) x = \gamma_o. \tag{6}$$

Once more, existence of a solution is not a problem but uniqueness is not guaranteed either.

2.5. Expected outcomes

For given preferences, costs distribution and quorum requirements, let γ_o^* and γ_s^* be the equilibrium strategies. γ_s^*/c and γ_o^*/c provide the expected value of the percentage of changers and conservatives, respectively, who will cast their vote. Therefore, the expected turnout rate, the expected percentages of 'Change' and 'Status Quo' votes and the margin of victory are easy to compute:

$$\begin{cases} E(Turnout) = \mu \frac{\gamma_s^*}{c} - (1-\mu)\frac{\gamma_o^*}{c} \ ; \\ E(\%\ Change) = \frac{\mu\gamma_s^*}{\mu\gamma_s^* + (1-\mu)\gamma_o^*}; \\ E(\%\ Status\ Quo) = \frac{\mu\gamma_o}{\mu\gamma_s^* + (1-\mu)\gamma_o^*}; \\ E(\%Margin) = \left|\frac{\mu\gamma_s^* - (1-\mu)\gamma_o^*}{\mu\gamma_s^* + (1-\mu)\gamma_o^*}\right|. \end{cases} \tag{7}$$

2.6. Probability of Change

Assuming that we know μ — the probability that each individual is a changer —, b, x and c — the benefits and the cost of voting — and the quorum rules, we can compute the equilibrium strategies γ_o^* and γ_s^*. Collecting this information, one can compute the probability of each election result, from which the probability that 'Yes' wins can easily be calculated. With this, we have a direct estimation of the effects of quorum rules and of its usefulness. For example, if a quorum rule is meant to bias the results for the status quo, then the quorum requirement is effective only if the probability of change decreases with its adoption. If the objective of the quorum is to obstruct active minorities from imposing their agenda, then the quorum does not meet its objectives unless there is a drop in the probability that the minority agenda wins.

Assume that there is no quorum. Let $\rho_n(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$ be the probability that, among the n individuals, v_s vote 'Yes' and v_o vote 'No', given their equilibrium strategies, γ_o^* and γ_s^*, and the voting cost distribution.[10] The probability that 'Change' wins is given by the sum of probabilities of all possible outcomes in which 'Yes' receives more votes than 'No':

$$P(Change|\ No\ quorum) = \sum_{v_s=v_o+1}^{n} \left(\sum_{v_o=0}^{\frac{n}{2}-1} \rho_n(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right). \tag{8}$$

If there is a quorum requirement, the reasoning is similar. The only difference is that the summation is restricted to outcomes that meet the quorum and, of course, that the equilibrium strategies will be quorum specific. In the case of an approval quorum of 25%, we have

[10] We do not derive the formulas here, because they are analogous to the derivations of $\rho(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$, $\rho_{aq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$ and $\rho_{pq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$. The only difference is that our population is now formed by n individuals instead of $n-1$.

$P(Change|\ Approval\ quorum) =$

$$= \sum_{v_s=\frac{n}{4}}^{n} \left(\sum_{v_o=0}^{\frac{n}{4}-1} \rho_{n,aq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right)$$

$$+ \sum_{v_s=v_o+1}^{n} \left(\sum_{v_o=\frac{n}{4}}^{\frac{n}{2}-1} \rho_{n,aq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right). \quad (9)$$

With a participation quorum, we have

$P(Change|\ Participation\ quorum) =$

$$= \sum_{v_s=\frac{n}{2}-v_o}^{n} \left(\sum_{v_o=0}^{\frac{n}{4}-1} \rho_{n,pq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right)$$

$$+ \sum_{v_s=v_o+1}^{n} \left(\sum_{v_o=\frac{n}{4}}^{\frac{n}{2}-1} \rho_{n,pq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right). \quad (10)$$

3. Scenario analysis

For now, we exclude overwhelming ex ante majorities, leaving the analysis of that case to section 4. It is known that the pivotal voter model does poorly at explaining large margins of victory (Coate et al., 2008). This is so because voters only have an incentive to vote if there is a reasonable chance that their vote is pivotal. An equilibrium with a large margin of victory is, therefore, rather difficult to obtain, precisely because the high margin implies that the probability of casting the decisive vote is close to zero. However, in situations where there are quorum requirements, the pivotal voter model is able to predict these types of outcomes.

Because there is no closed form solution to this problem, we have to rely on numerical methods to find solutions. To simulate our model, we fix the number of eligible voters. For computational reasons we consider only $n = 200$ voters and, each time we have to compute a binomial probability, with parameters p and N, we use a Normal approximation if $pN > 5$ and $(1-p)N > 5$, and the exact binomial probabilities otherwise. To look for equilibria, we start with a thin grid search. After that, using a standard algorithm, we individually iterate from the most promising candidates.

We have to choose the parameter values for $b, x, \mu,$ and c. For our benchmark, we consider that changers and conservatives have equally strong feelings about the issue,

$b = x = 22.5$. The choice of c is irrelevant, as we can see in the system of equations (1) and (2), because only the ratios γ_o/c and γ_s/c matter. We consider $c = 1$.

3.1. Very competitive referendum

We consider a scenario as competitive as possible, with equal prior chances, $\mu = 0.5$. We chose the values for b and x in such way that, with no quorum requirements, the participation rate is close to 75%. Therefore, without quorum requirements, we have a competitive election with a relatively high turnout rate.

In Figure 2, we illustrate how quorum rules distort the incentives that each person has to vote. The graphs on the left represent the incentives for a conservative in the presence of the different quorum rules (no quorum, participation quorum and approval quorum). On the right, we have the same for changers.

Consider the case of a conservative, in a no-quorum environment (graph on the top left). We know from the previous section that his or her expected benefit of voting is $\sum_{v=0}^{\frac{n}{2}-1}[\rho(v+1, v; \gamma_s, \gamma_o, c)]x$. To calculate the net benefit, one has to subtract the costs; however, costs are different for each individual. So we calculate the net benefit for the voter that is in the margin, i.e. $\gamma_o = c_i$. Thus, what we call the 'marginal net expected benefit of voting' is simply $\sum_{v=0}^{\frac{n}{2}-1}[\rho(v+1, v; \gamma_s, \gamma_o, c)]x - \gamma_o$.

Note that an equilibrium implies that γ_o and γ_s are such that the marginal net expected benefit of voting is zero both for conservatives and changers. The exception to this rule is, naturally, corner solutions, in which case it is possible to have a negative/positive benefit and $\gamma = 0/\gamma = 1$.

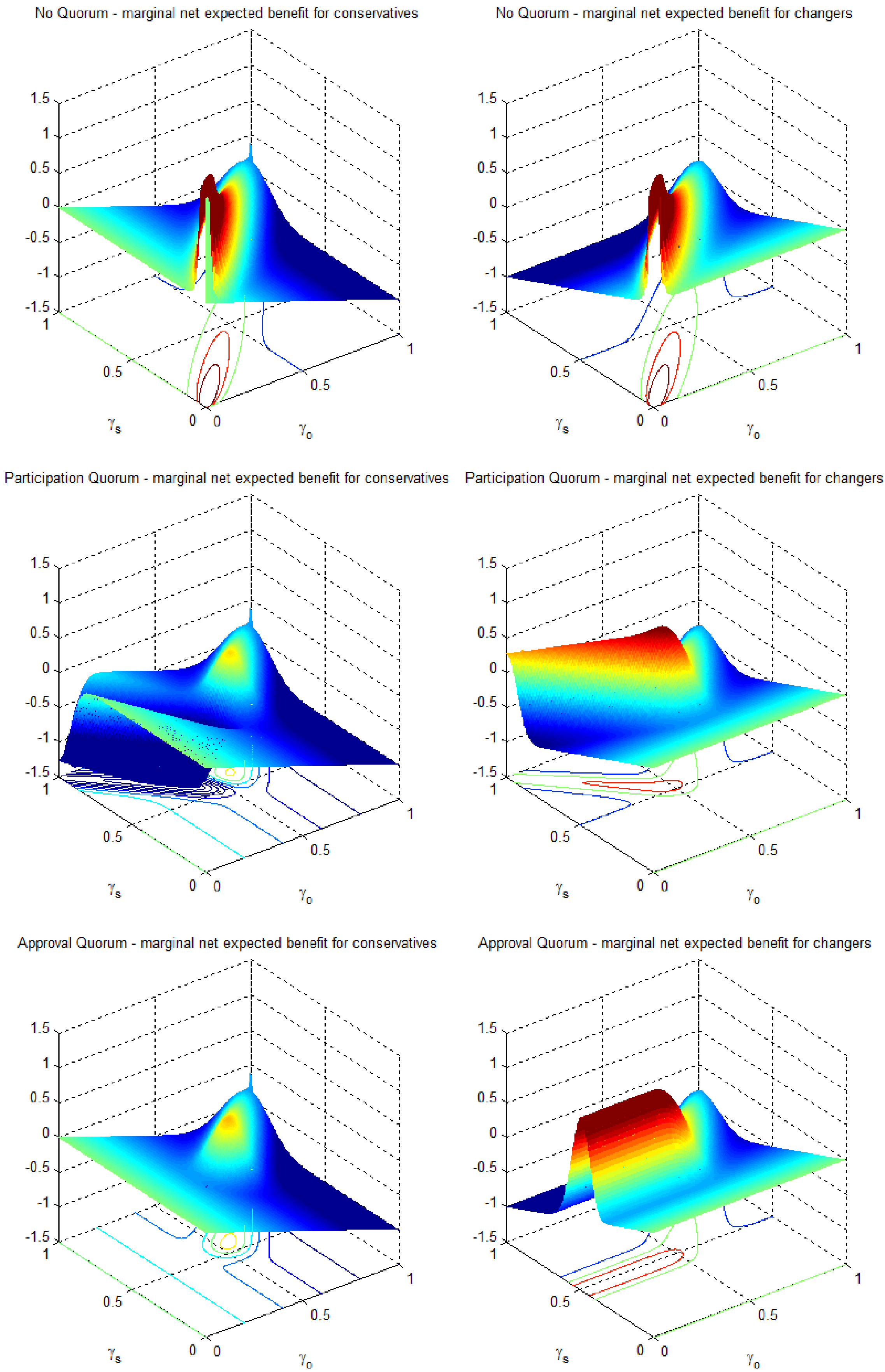

Figure 2: Marginal net benefits of voting

In the case of no quorum, the incentives are quite intuitive. As long as γ_o is similar to γ_s and turnout is not too high, the net benefits are positive.[11] With quorum requirements, things are more complicated and asymmetric. Consider the case of a participation quorum. There is a long valley in the incentives of a conservative, which corresponds to the negative incentives associated with reaching the quorum when change has the majority of votes. Any equilibrium in that valley must be a corner solution, with conservatives deserting the ballots. With changers, incentives are asymmetric. Instead of a valley, we have a long hill of warm colors. No equilibrium is possible in that region. However, the graphs suggest two possible interior solutions. We have green regions for high values of γ_o and γ_s, but we also observe a green region for smaller values, when conservations are moving from the valley to the peak of the mountain. Additionally, there are a few other possible equilibria. One is no-show equilibrium (when $\gamma_o = \gamma_s = 0$, net incentives are zero). Other equilibria are also associated with $\gamma_o = 0$. One is $\gamma_o = 0$ and $\gamma_s = 1$. Net incentives are negative for conservatives and positive for changers. Of course, it is also possible that $\gamma_o = 0$ and γ_s is high but smaller than one, corresponding to the green region, when net incentives are zero.

Finally, in the bottom, we have the approval quorum case. From the perspective of a conservative, it looks a lot like the participation case, without the dark blue valley. From a changer's perspective, it is also similar to the participation quorum, but now the hill is associated to $\gamma_s = 0.5$, which is necessary to meet the approval quorum requirement. So one can expect similar interior solutions. For corner solutions the rationale is slightly different. For $\gamma_o = 0$, conservatives always have zero net benefits from voting. In the changers graph, we observe three different green regions associated with $\gamma_o = 0$. One is associated with $\gamma_s = 0$. The other two are just the opposite sides of the same hill, which peaks for . $\gamma_s = 0.5$

Table 1 confirms these predictions for our benchmark scenario. For the next tables, rows representing scenarios in which 'Change' has more than 55% chance of winning have a white background. Scenarios in which the 'Status Quo' has more than 55% chance to prevail have a black background and, when the chances are even (neither side has more than 55% chance of winning) the background is in gray. The probability that

[11] Color green corresponds to net benefits close to zero. Warmer colors imply positive values and colder colors imply negative values. We truncated the positive values at 1.5, which occurred for very low values of γ_o and γ_s, and only in the no quorum case, to avoid distortions in the graph. For example, if $\gamma_o = \gamma_s = 0$ the net benefit for a changer is b=22.5.

'Change' wins is given in the second to the last column. In the last column, we have the probability of change conditional on having $P_C = \mu$, where P_C is the realized proportion of the electorate that favors 'Change'.[12]

Table 1: Equilibrium outcomes when the population is evenly divided

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total votes)	Expected Percentage of Votes for Change (total registered voters)	Probability that Change Wins	Probability that Change Wins given that $Pc = \mu$
No quorum	eq1	73,8%	74,0%	73,9%	50,1%	37,0%	48,9%	47,9%
Participation Quorum	eq1	73,8%	74,0%	73,9%	50,1%	37,0%	48,9%	47,9%
	eq2	53,0%	62,6%	57,8%	54,2%	31,3%	79,5%	89,7%
	eq3	0,0%	100,0%	50,0%	100,0%	50,0%	52,8%	100,0%
	eq4	0,0%	94,1%	47,0%	100,0%	47,0%	22,1%	1,0%
	eq5	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%
Approval Quorum	eq1	73,8%	74,0%	73,9%	50,1%	37,0%	48,9%	47,9%
	eq2	48,1%	59,1%	53,6%	55,1%	29,6%	83,3%	92,2%
	eq3	0,0%	57,9%	29,0%	100,0%	29,0%	90,7%	95,6%
	eq4	0,0%	39,8%	19,9%	100,0%	19,9%	4,6%	2,4%
	eq5	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%

n = 200, b = 22.5, x = 22.5, μ = 0.50, participation quorum=50%, approval quorum=25%

In the case of no quorum, 73.8% of conservatives cast their vote. The percentage among the changers is similar.[13]

Under a participation quorum, several equilibria emerge. Two were to be expected. The high participation one (eq. 1), is similar to the equilibrium with no quorum. This is so because with such a high participation rate, quorum requirements are not a binding constraint (or the probability of being so is almost zero). Another possible equilibrium is simply the no-show equilibrium (eq. 5). Nevertheless, there are also some intermediate cases, which confirm our previous graphical analysis.

In equilibrium 2, conservatives vote less than changers. The turnout rate is smaller than in the case of no quorum, but it is still enough to reach the participation requirement.

[12] Although related, the two last columns do not provide the same information. Reasoning in terms of large numbers, the second to the last column gives us the proportion of times that 'Change' wins if the voting game is repeated infinitely many times. The last column gives the same proportion but restricted to the subset of games in which expected proportion of changers is equal to the actual proportion of changers. This distinction is not very relevant in this scenario, but it will be informative in the next scenarios. In those cases, it is informative to distinguish between an expected majority and an actual majority.

[13] Actually, it is slightly more because of the asymmetry we have introduced by assuming that for conservatives a tie was enough to win the election.

This equilibrium provides an excellent illustration of how the quorum restrictions distort the incentives that voters face. Without any quorum, these participation rates among conservatives and changers (53% and 62.6% respectively) could never be equilibrium participation rates. Conservatives would have an incentive to increase their participation. They do not do so precisely because an increase in their participation increases the probability that the quorum is met (this is illustrated by the negative term in equation 6). A participation quorum can and is sometimes described as introducing a bias for the 'status quo', in order to protect an equilibrium that the society achieved. As we saw in Figure 1.c, this is thought to be accomplished by increasing the 'status quo' region. However, what this exercise shows is that, in some cases, a quorum may be a blessing in disguise to changers. If equilibrium 2 prevails, change will occur with a probability of 79.5%.

Equilibria 3 and 4 (and 5) show more radical results, with very important consequences. In these cases, the abstention rate among opponents is 100%. This way, abstention is an almost functional equivalent of a 'No' vote and, as a result, although the 'change' option receives 100% of the votes, the 'status quo' wins with almost 50% chance in equilibrium 3[14] and with 80% chance in equilibrium 4. According to equilibrium 3, changers' participation rate is 100%, a corner solution. This is so because of the specific parameter values we chose. With other parameters we one have an interior solution with a very high participation rate, but less than 100% (e.g. see Table 3). One can argue that Equilibrium 4 (the intermediate case among the equilibria with 100% abstention rate among conservatives) is unstable and, therefore, very unlikely. Note that given that conservatives do not participate, the final outcome depends on the changers' mobilization rate. And for any participation rate of changers smaller than 94%, several voters would be unhappy with their own strategies and would rather abstain (the net benefits are negative), while for a participation above that threshold, several abstainers would rather vote (the net benefits are positive). The no-show equilibrium (equilibrium 5), however, is stable, because it would be needed a huge coordination among changers to move to a more favorable equilibrium.

[14] The information given in the last column tells us that in this equilibrium 'change' wins 100% of the times. This happens because in this column we are assuming that exactly 50% of the electorate is in favor of change and each supporter will vote with 100% probability, while conservatives do not show up in the polls. Therefore, victory is guaranteed. This is the only case where the last two columns give such disparate predictions

These sorts of outcomes, with zero participation rates among conservatives, are far from being a mere theoretical curiosity. Consider, for example, the case of the Italian 'abrogative referenda' — called to wholly or partially repeal an existing law. The requirement that over half of the electorate needs to participate in order for results to be valid has given those who support the law two alternative courses of action: to support the 'No' option or, instead, to simply abstain from voting in order to render the result null and void (Uleri, 2002). As a result, of the 49 issues decided by referendum in Italy from 1987 to 2007, 24 failed to meet the required turnout quorum, displaying extremely low levels of support for the 'No' option among those who actually voted (Kaufmann et al., 2008, p. 218). Many other contemporary and historical examples of massive demobilization by status quo supporters under turnout quorum requirements can be found, including the referenda held under the German Weimar republic, several referenda held in Eastern Europe since the early 1990s, and others.[15]

The approval quorum has similar effects, although not identical. Approval quorum equilibria 1, 2 and 5 are similar to the participation quorum equilibria 1, 2 and 5. The main distinction is that there are two different consequences when opponents decide not to participate. In one of them, turnout among supporters is almost 58%, while in the other it is less than 40%. In one case, the quorum is met with a probability of 90%. In the other case, the 'status quo' prevails in 95% of the time, generating the same awkward situation described before.[16] Again, real world examples of massive demobilization on the part of conservatives under approval quorum restrictions can be easily found, including the two constitutional referenda held under the Danish 1920 Constitution (Suksi, 1993, p. 211; Svensson, 1996, pp. 38-40) and several referenda held at the state and municipal level in Germany (Verhulst and Nijeboer, 2007, pp. 19-21) and at the national level in Eastern Europe.[17]

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[15] The two referenda held under the German Weimar Republic, under a 50% turnout quorum rule, resulted in overwhelming majorities voting 'Yes' but very low levels of turnout and the invalidation of the results (Suksi 1993: 95). In Slovakia, for example, where a 50% turnout requirement for validity remains in force, only one of the six referenda held since 1994 has surpassed that threshold and, in five of them, the share of the vote for 'Yes' was equal or above 87 per cent. And in Colombia, an October 2003 referendum pushed by President Alvaro Uribe on no less than 15 different issues was the object of a campaign for abstention organized by trade unions and opposition parties, and turnout levels ultimately failed to meet the 25% requirement for validity (IDEA 2008: 182).

[16] As before, one can argue that Equilibrium 4 is not likely to occur.

[17] In three of four referenda held in Hungary since the mid-1990s, under a 25% approval quorum requirement, percentages of voters for 'Yes' have been above 80%. In Latvia, under a 50 per cent approval quorum rule, the August 2008 referendum on the introduction of the possibility of dissolving parliament by popular vote resulted in a 97 per cent majority for 'Yes', but only a 42% turnout.

3.2. What if majority is for the status quo?

In this example, the only difference is that we now assume that $\mu = 0.45$. This means that the expected percentage of conservatives among potential voters will be 55%. Therefore, a referendum outcome that mirrors the majority must be one in which the 'No' is expected to win.

Table 2: Equilibrium outcomes when there is a 55% majority in favor of the status quo

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total votes)	Expected Percentage of Votes for Change (total registered voters)	Probability that Change Wins	Probability that Change Wins given that *Pc* = *μ*
No quorum	eq1	58,8%	63,1%	60,8%	46,7%	28,4%	22,1%	11,0%
Participation Quorum	eq1	59,2%	63,6%	61,2%	46,8%	28,6%	22,6%	11,3%
	eq2	47,4%	68,5%	56,9%	54,2%	30,8%	78,5%	88,9%
	eq3	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%
Approval Quorum	eq1	58,9%	63,2%	60,8%	46,7%	28,4%	22,2%	11,1%
	eq2	0,0%	63,8%	28,7%	100,0%	28,7%	89,4%	96,0%
	eq3	0,0%	44,7%	20,1%	100,0%	20,1%	5,4%	2,5%
	eq4	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%

n = 200, *b* = 22.5, *x* = 22.5, *μ* = 0.45, participation quorum = 50%, approval quorum = 25%

Table 2 gives us quite striking results that can only be labeled as paradoxical. If a quorum requirement is absent, the 'No' option is expected to win with a comfortable margin of almost 7 percentage points. In turn, this implies that the status quo prevails with a probability of 77.9%, or 89% if we take as given that exactly 55% of the electorate is against changing. If there is a quorum requirement, no matter which type, the first equilibrium is in the neighborhood of the 'no quorum' equilibrium.

However, the expected outcome is reversed in two equilibria under quorum rules. Under a participation quorum, the second possible equilibrium implies a smaller percentage of conservatives voting. This happens because a conservative vote has two conflicting consequences: on the one hand, it contributes to the status quo majority, but on the other hand, it helps 'Change' to reach the quorum. The implication is that if a conservative is afraid that a majority of votes supports the proposal, his or her best option may be to abstain, rather than voting. Under this equilibrium, 68.5% of changers will vote, which is enough to give them a solid majority (8.4 percentage points ahead).

Under an approval quorum requirement, the second and third equilibria involve the total abstention of conservatives, an outcome we had already observed under competitive conditions. If supporters are able to coordinate to show up in the polling stations in big numbers (equilibrium 2), they will win the referendum, imposing their will upon a majority of conservatives. Note that the approval quorum requirement is that 25% of the electorate votes ‘Yes’ and equilibrium 2 implies that almost 29% of the electorate votes ‘Yes’.

Therefore, one of the arguments for the quorum requirements loses its strength even further. The quorum requirement, instead of defending the ‘status quo’, may actually be working the other way around, creating a bias for ‘change’. To be harmless, the equilibrium under the quorum requirement should be in the neighborhood of the ‘no quorum’ equilibrium, but, of course, in this case it would just be simpler to forego a quorum.

3.3. What if majority is for change?

We consider now the reverse scenario: $\mu = 0.55$. An expected majority of people is for change. Which of the three systems is the best to reflect these choices? Looking at Table 3, once again, we confirm that quorum requirements introduce equilibria that do not match voters’ preferences.

Table 3: Equilibrium outcomes when there is a 55% majority in favor of the change

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total votes)	Expected Percentage of Votes for Change (total registered voters)	Probability that Change Wins	Probability that Change Wins given that $Pc = \mu$
No quorum	eq1	64,5%	60,5%	62,3%	53,4%	33,3%	76,3%	87,9%
Participation Quorum	eq1	0,0%	95,3%	52,4%	100,0%	52,4%	77,8%	99,3%
	eq2	0,0%	84,7%	46,6%	100,0%	46,6%	18,6%	4,7%
	eq3	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%
Approval Quorum	eq1	64,5%	60,5%	62,3%	53,4%	33,3%	76,3%	87,9%
	eq2	56,7%	55,7%	56,1%	54,5%	30,6%	81,9%	91,4%
	eq3	0,0%	53,0%	29,2%	100,0%	29,2%	91,7%	95,4%
	eq4	0,0%	35,9%	19,7%	100,0%	19,7%	4,0%	2,3%
	eq5	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%

***n* = 200, *b* = 22.5, *x* = 22.5, *μ* = 0.55, participation quorum = 50%, approval quorum = 25%**

Both the participation and approval quorum equilibria show two possibilities where, under a majority of supporters, the status quo wins, and they involve the massive demobilization of conservatives. The equilibria associated with the participation rule are particularly striking, as all the equilibria that we found have one common characteristic:

the desertion of people who favor the status quo. This makes sense: as it is very unlikely that conservatives will have the majority of votes, by showing up they would help changers to meet the quorum requirement. Thus, they desert the polls and, in some cases, are able in this way to defeat a majority of the population.[18]

Ex ante measures of preferences in concrete cases of referenda are highly fallible, resulting invariably from a (often scarce) number of opinion polls whose results may already incorporate the effects of quorum themselves (particularly in terms of the likelihood to vote). However, there is a clear example of the phenomenon we have just described in the Italian June 2005 referendum on the abolition of restrictions to in vitro fertilization and embryo research in Italy.[19] If successful, the referendum would have lifted a ban on the freezing of embryos, permitted their screening to identify genetic defects and allowed more than three to be implanted, eliminated a passage in the existing law that gives embryos the same rights as parents, and repealed the prohibition of homosexual couples and single people from having children via assisted fertility. Throughout 2004 and early 2005, all opinion polls on the subject indicated that a majority of Italians was supportive of in vitro fertilization and therapeutic cloning and that, in spite of a large number of undecided voters, a majority of voting age respondents were in favor of changing the existing legislation.[20] These polls, by bringing the clear perception that the 'Yes' vote might win, seemed to have played no small role in the reaction from the Catholic Church: the Conference of Bishops, led by Cardinal Ruini, engaged in a vigorous campaign to prevent the abrogation of the existing law, by appealing to abstention. Pope Benedict XVI publicly backed the boycott campaign, which was joined by the Northern League and the Popular Alliance parties, as well as some figures of the governing *Forza Italia*.[21] In the end, in spite of

[18] Note that one could argue that both the participation quorum equilibrium 2 and the approval quorum equilibrium 4 are unstable. However, the no show equilibrium is stable and mass demobilization is, therefore a viable course of action.

[19] In particular, it is a good illustration of the participation quorum equilibrium 1 or 2.

[20] In September 2004, a Eurispes poll showed that 65% of Italians were in favour of in vitro fertilization and 64% in favour of therapeutic cloning. In the same month, an Swg poll showed a majority of 54 per cent supporting the abrogation of the existing legislation and 34% against it, while in January 2005, a TNS poll showed that only 11% of respondents leaned towards or had already decided against the abrogation of the law, with 35% were totally or partially in favour of repealing it, with the remaining unsure. See also the Ispo April 27-28 poll and the Istituto Piepoli June 6th poll. All results collected from Angus-Reid Global Monitor, at: http://www.angus-reid.com/polls.

[21] 'When opinion polls showed that a win for the 'yes' vote seemed likely bishops throughout the land launched a vigorous campaign urging Italians to boycott the referendum. Pope Benedict XVI, who in addition to being pontiff is also bishop of Rome, publicly backed the boycott campaign.' – Roland

the ‘Yes’ votes on the different issues reaching shares of the valid vote that ranged from 77 to 88%, turnout was only 26%, and the law stayed in force. In other words, with a majority for change, status quo ended up prevailing under quorum rules.

3.4. Implications

Under the different scenarios examined so far, quorum requirements may produce a series of disturbing consequences. First, the contrast between the outcomes without quorum rules and several equilibria under such rules reveals what in the literature became known as the ‘No-Show paradox’: while both the quorum requirements would be reached in the absence of quorum rules, there are equilibria where, precisely because of the presence of such quorum restrictions, they end up not being met. As Aguiar-Conraria and Magalhães (forthcoming) show, this possibility is empirically very plausible. Second, there are equilibria where, instead of introducing an alleged bias for ‘status quo’, quorum restrictions actually favor supporters of change. Third, several equilibria yield the result where, in the presence of quorum restrictions, minorities end up imposing their will on majorities.

In fact, this last possibility becomes even more real when we consider the additional consequence of a very frequent sort of equilibrium in our analysis: the mass demobilization of status quo supporters. To the extent that turnout and abstention are observable, this creates conditions under which voting choices on the part of the electorate may, in practice, cease to be protected by a veil of secrecy. Weimar provides a textbook case. The 1926 referendum over the confiscation of princely properties, held under a 50% turnout quorum rule, resulted in a very large majority supporting that confiscation (well over 90%) but a turnout level of 36%, rendering the result invalid. This was largely the result of a campaign for abstention organized by landowners and organizations linked to the Bavarian People’s Party and the National Socialists. Systematic voter intimidation on their part – what the supporters of the measure called *Wahlterror* – lead many to stay at home, particularly in the rural areas (West, 1985, p. 247; Verhulst and Nijeboer, 2007, p. 82). This failure, repeated in the 1932 referendum, was one of the most visible signs of the latent crisis of the institutions of Weimar Germany, and the notion that the quorum rule had ‘impeded the legitimate expression of majority will’ was largely responsible for a long-lived legacy of distrust regarding direct

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Flamini, ‘Church blocks Italian referendum’, *United Press International*, June 15th, 2005, at: http://www.religiousconsultation.org/News_Tracker/Church_blocks_Italian_referendum.htm.

democracy in Germany (West, 1985). Similar problems occurred in the Danish 1920 and 1939 referenda (Suksi, 1993, p. 94 and p. 211). And there are good reasons to believe that such concerns about the lack of voter anonymity are not a mere historical curiosity. A few days before the Italian 2005 referendum, a woman living in a small town in the Central Italian region of Lazio reported how the lack of anonymity of the vote in the forthcoming referendum was resulting in less than subtle forms of social pressure.[22]

4. Can a small noisy minority be decisive?

Campbell (1999), using a standard pivotal model, showed that it is possible that small minorities with very strong feelings about an issue end up imposing their view on an apathetic majority. He considers two different normative criteria to check if this outcome is socially desirable. According to a 'democratic criterion' (1), an outcome is desirable only if the majority of the population prefers that same outcome; according to an 'economic criterion' (2), one should sum all the individuals' utility and choose the one that provides the greater total utility. Campbell shows that both criteria will be frequently violated in the presence of a small very active minority.

As pointed out by Qvortrup (2005), the possibility of this type of outcome is one of the most common arguments for the existence of quorum requirements: they attempt to avoid the problem in which, in situations of high voter apathy, 'a special group of committed citizens may well take advantage of the situation by trooping to the polls in great numbers while the majority of the voters stay at home' (p. 173). Unfortunately, as we will show, a quorum is not a good remedy for this situation. We will provide two examples. In one of them, a very active small minority group of changers will receive precious help from quorum rules in order to impose their will. The second scenario is similar, but the active minority is for the 'status quo' instead.

4.1. Active minority for 'Change'

We consider $\mu = 0.30$, meaning that the expected percentage of the population for change is 30%, while an expected overwhelming majority of 70% is against. Following

[22] 'In Capranica, a town of 6000 people, voting will be difficult. The local church organized groups of volunteers to intimidate the town's people to prevent them from voting. My town is not unique. There are several municipalities where anonymity does not exist. People are controlled, their votes, their actions and whereabouts. The degree of distress, especially among women, is enormous.' At: http://ecumenici.altervista.org/html/pivot/entry.php?id=233.

Campbell (1999), we introduce a twist. We assume that the minority is strongly in favor of the proposal while the majority does not have such strong feelings for the subject. This is captured by assuming that $b = 30$, while $x = 15$, so that the utility of winning the referendum is twice as much for the supporters than for the opponents. Note that, according to both the democratic and the economic criterions, it is socially desirable that the 'status quo' prevails.

Table 4: Equilibrium outcomes when there is a very active minority (30%) for change

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total votes)	Expected Percentage of Votes for Change (total registered voters)	Probability that Change Wins	Probability that Change Wins given that *Pc* = *μ*
No quorum	eq1	45,0%	94,9%	60,0%	47,5%	28,5%	27,6%	14,3%
Participation Quorum	eq1	45,4%	96,3%	60,7%	47,6%	28,9%	28,7%	15,2%
	eq2	38,0%	100,0%	56,6%	53,0%	30,0%	70,6%	85,5%
	eq3	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%
Approval Quorum	eq1	45,1%	95,2%	60,1%	47,5%	28,6%	27,9%	14,5%
	eq2	34,3%	95,7%	52,7%	54,4%	28,7%	78,6%	93,6%
	eq3	0,0%	94,6%	28,4%	100,0%	28,4%	87,3%	99,9%
	eq4	0,0%	67,9%	20,4%	100,0%	20,4%	6,4%	0,7%
	eq5	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%

n = 200, b = 30, x = 15, μ = 0.30, participation quorum = 50%, approval quorum = 25%

In this scenario, under the no quorum equilibrium, the 'status quo' wins with a probability of more than 70%. The same happens in equilibrium 1, under the participation or under the approval quorum. However, in the presence of a quorum requirement, other equilibria emerge under which 'Change' is expected to win. In the case of a participation quorum, one of the equilibria involves less conservatives voting and a mass run to the polls by changers. Change wins with a probability of 70%. With an approval quorum, 'Change' wins under two equilibria. One of them involves partial desertion from conservatives, while another total desertion. In one case, 'change' wins with a probability of almost 80%. In the other case, there is a probability of more than 87% that 'change' wins.

Therefore, in this scenario of a very active minority for change, a quorum requirement, instead of protecting the majority, may cause the opposite: to make the active minority more powerful.

4.2. Active minority for the 'Status Quo'

We now consider the reverse scenario. There is a noisy minority for the status quo and an apathetic majority for change: $\mu = 0.70, b = 15, x = 30$. In this scenario, the vast majority of citizens support 'Change', which, without quorum requirements, wins with a probability of 70%. According to both of Campbell's criteria, the socially desirable outcome is 'change'. However, as we can see in table 5, both quorum types generate equilibria in which the preferences of the conservative minority are likely to prevail.

Table 5: Equilibrium outcomes when there is a very active minority (30%) for the status quo

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total votes)	Expected Percentage of Votes for Change (total registered voters)	Probability that Change Wins	Probability that Change Wins given that *Pc* = *μ*
No quorum	eq1	96,6%	46,1%	61,2%	52,7%	32,2%	70,5%	83,9%
Participation Quorum	eq1	0,0%	73,9%	51,7%	100,0%	51,7%	70,8%	77,4%
	eq2	0,0%	67,8%	47,4%	100,0%	47,4%	25,6%	20,2%
	eq3	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%
Approval Quorum	eq1	96,5%	46,0%	61,2%	52,7%	32,2%	70,6%	84,0%
	eq2	86,4%	43,1%	56,1%	53,8%	30,1%	77,1%	89,0%
	eq3	0,0%	41,1%	28,8%	100,0%	28,8%	89,7%	91,7%
	eq4	0,0%	28,6%	20,0%	100,0%	20,0%	4,9%	3,9%
	eq5	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%

***n* = 200, *b* = 30, *x* = 15, *μ* = 0.70, participation quorum = 50%, approval quorum = 25%**

5. Can quorum rules enhance turnout?

In Table 4, we can see that the first equilibrium with quorum implies a slightly higher turnout than with no quorum. Nevertheless, this effect is very marginal and does not change the general impression that quorum requirements promote abstention. Herrera and Mattozzi (2010) also concluded that quorum requirements can promote abstention. However, according to the empirical results of Aguiar-Conraria and Magalhães (forthcoming), while a participation quorum has detrimental effects on turnout, an approval quorum, on average, has negligible, even possibly positive, effects on turnout rates. Is our pivotal voter model compatible with these empirical results?

We provide a final example, in which the approval quorum may significantly increase turnout.[23] We consider a scenario in which the public opinion is divided ($\mu = 0.50$) and changers have weaker feelings about the issue. This implies that, under no quorum

[23] We were not able to find a similar example for the participation quorum.

distortions, the ‘Status Quo’ is expected to win, which is the socially desirable outcome according to Campbell’s economic criterion.

Table 6: Equilibrium outcomes when the population is evenly divided but conservatives have much stronger feelings

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total votes)	Expected Percentage of Votes for Change (total registered voters)	Probability that Change Wins	Probability that Change Wins given that $Pc = \mu$
No quorum	eq1	44,5%	34,1%	39,3%	43,4%	17,1%	10,9%	5,6%
Approval Quorum	eq1	56,7%	49,9%	53,3%	46,8%	25,0%	22,8%	14,0%
	eq2	0,0%	55,9%	28,0%	100,0%	28,0%	84,5%	90,3%
	eq3	0,0%	41,5%	20,8%	100,0%	20,8%	8,4%	5,2%
	eq5	0,0%	0,0%	0,0%	NA	0,0%	0,0%	0,0%

n = 200, b = 15, x = 22.5, μ = 0.50, participation quorum = 50%, approval quorum = 25%

In this situation, the existence of an approval quorum may fight abstention. The strategic reasoning is clear. Under the no quorum benchmark, conservatives will win the referendum. This happens because conservatives care more deeply about the issue, not because they were in majority. If an approval quorum is introduced, the expected benefit of voting for a changer increases, because it increases the probability of being pivotal (the voter has two chances of being pivotal: his or her vote may be decisive to reach the quorum or to reach majority). Therefore, there is an increase in the participation of the changers. This, in turn, implies a reaction from the conservatives. If they want to win the election, they have to increase their participation too. Overall, participation of members of both groups increases significantly. As in can see in Table 6, in equilibrium 1 under an approval quorum, the ‘status quo’ still wins and turnout increases by 14 percentage points.[24] As in previous examples, conservatives desert the ballots in other equilibria.

6. Consumption or expressive voters

So far, we have assumed that all voting is instrumental. Voters only derive benefits from voting to the extent that their vote may influence the referendum outcome. A voter facing a negative voting cost or expressive benefit will vote even if he or she believes that the probability of casting the decisive vote is zero. Typically, these negative costs or expressive benefits are added to the model to avoid the paradox of voting: if one

[24] It would be easy to construct a similar example in which change wins.

assumes that all voting is instrumental, given that the chance of casting a decisive vote is too small to justify any realistic private individual benefit, voter turnout should be considerably smaller than observed in reality. Therefore, when voters are not pivotal, the act of voting has been regarded as independent of probabilistic considerations about causal consequences of voting and related to, for example, a 'sense of civic duty' or 'relational goods' (Feddersen, 2004), or voters have been described as voting expressively, to show support to their favorite side (Brennan and Buchanan, 1984; Glazer, 1987; Brennan and Hamlin, 1998) or more particularly they use their vote as a means of expressing their identity (Hillman, 2010).[25] Palfrey and Rosenthal (1985) showed that negative voting costs (or expressive benefits) rule out of the possibility of mixed-pure strategy equilibria. In mixed-pure strategy equilibria, voters are divided into three groups: pure randomizers, abstainers, and those who vote with probability one. They showed that, if we consider only symmetric equilibria – the ones on which we have focused – then the theorems they derive are applicable essentially in the same way with or without consumption voters (1985, appendix B, p. 75). However, we can examine what happens when we repeat the computational simulations with different proportions of consumption or expressive voters.[26]

We allow for expressive voting by allowing negative voting costs and changing the cost distribution function. In this section we assume $c_i \sim U[-\bar{c}, 1]$. The proportion of consumption voters is given by $\bar{c}/(1+\bar{c})$. This change in the model implies that among the s supporters, the probability that v_s of them will vote is now given by $V(v_s) = \binom{s}{v_s}\left(\frac{\bar{c}}{1+\bar{c}} + \frac{1}{1+\bar{c}}\gamma_s\right)^{v_s}\left(1 - \frac{\bar{c}}{1+\bar{c}} - \frac{1}{1+\bar{c}}\gamma_s\right)^{s-v_s}$. Similarly, the probability that, among the other $n-1-s$ registered voters, v_o will vote 'No' is now given by $V(v_o) = \binom{n-1-s}{v_o}\left(\frac{\bar{c}}{1+\bar{c}} + \frac{1}{1+\bar{c}}\gamma_o\right)^{v_o}\left(1 - \frac{\bar{c}}{1+\bar{c}} - \frac{1}{1+\bar{c}}\gamma_o\right)^{n-1-s-v_o}$. By continuity, it is obvious that, if $\bar{c}$ is close to zero, its effects will be negligible. Therefore, we focus on higher values.

But which values are appropriate? There is no agreement on the part of an electorate that might be consumption or expressive voters. A large part – about half – of voters in national electorates in several countries tends to describe voting as a civic duty in surveys. Such responses reflect a desire to indicate adherence to social norms (Blais,

[25] For an application, see for example Fidrmuc et al. (2009).
[26] For a similar approach see, for example, Castanheira (2003).

2000, p. 112); the responses can also expressively indicate confirmation of an identity as a socially responsible citizen (Hillman, 2010).

As Jankowski (2002) notes, large numbers of expressive consumption voters contradicts facts about voter turnout. For example, both in the 2004 and 2009 European Parliament elections, turnout in several countries was below 30% or even 20%. The same occurs with local elections in the United States (Hajnal and Lewis, 2003) and, in a recent off-cycle ballot initiative in California (in May of 2009), turnout of eligible voters was as low as 20.8%. In fact, policy makers of several different countries seem to agree that the number of consumption voters is modest: otherwise, high turnout rates were guaranteed and there would be no need for quorum rules. All this suggests that something like one-quarter of the electorate may be a good upper bound for an estimate of the mass of consumption voters. Thus, we modify our benchmark scenario (Table 1) by admitting, in Table 7, that the proportion of consumption voters may be 25% ($\bar{c} = 1/3$). We also estimated the results for 35% ($\bar{c} = 7/13$), and 50% ($\bar{c} = 1$)[27] but report them only in the text.

Table 7: Equilibrium outcomes when 25% of the electorate faces negative costs

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total votes)	Expected Percentage of Votes for Change (total registered voters)	Probability that Change Wins	Probability that Change Wins given that $Pc = \mu$
No quorum	eq1	78,7%	78,8%	78,7%	50,0%	39,4%	48,8%	47,4%
Participation Quorum	eq1	78,7%	78,8%	78,7%	50,0%	39,4%	48,8%	47,4%
	eq2	50,9%	63,7%	57,3%	55,6%	31,8%	86,2%	94,7%
	eq3	0,0%	100,0%	50,0%	100,0%	50,0%	52,8%	100,0%
	eq4	0,0%	93,9%	46,9%	100,0%	46,9%	21,2%	0,9%
	eq5	25,0%	25,0%	25,0%	50,0%	12,5%	0,0%	0,0%
Approval Quorum	eq1	78,7%	78,8%	78,7%	50,0%	39,4%	48,8%	47,4%
	eq2	44,6%	59,6%	52,1%	57,2%	29,8%	90,2%	96,7%
	eq3	25,0%	59,0%	42,0%	70,2%	29,5%	93,2%	97,4%
	eq4	25,0%	37,3%	31,1%	59,9%	18,6%	1,5%	0,6%
	eq5	25,0%	25,0%	25,0%	50,0%	12,5%	0,0%	0,0%

$n = 200$, $b = 22.5$, $x = 22.5$, $\mu = 0.50$, participation quorum = 50%, approval quorum = 25%

Table 7 tells us that the existence of 25% of consumption voters does not change the main conclusions of Table 1. The equilibrium with no quorum has a higher participation

[27] One can argue that expressive voters have no particular interest in expressing their support in elections regarded as less important, such as parliamentary elections and some referenda. Therefore, it is possible that the number of 25% underestimates the percentage of expressive voters.

rate, as expected. This equilibrium is replicated with quorum requirements. Equilibrium 2, both in the participation of approval quorum cases, is also similar to the equilibrium 2 that we have found in Table 1.

In Table 1, equilibria 5 corresponded to the no show equilibrium: if nobody else was showing up in the polls, then each voter had no incentive to vote, given the probability zero of changing the outcome. The new equilibria 5 in Table 7 have an expected turnout of 25%, which corresponds to the expected percentage of consumption voters.

Equilibria 3 and 4 are the most informative in this table because they reveal how the incentives are different under different quorum requirements. In Table 1, we saw conservatives deserting the ballots, no matter which specific quorum was in effect. In Table 7, this desertion only occurs under the participation quorum. With an approval quorum, equilibria 3 and 4 show that conservative consumption voters do show in the polls, while they prefer to abstain if there is a participation quorum. This happens because conservatives must add the cost of helping 'Change' to reach the quorum to the negative cost of voting. With an approval quorum, however, their decision has no impact on the quorum: if they enjoy voting, they vote, if voting is costly, they don't. Thus, Table 7 helps illuminating equilibria 3 and 4 found in Table 1. With an approval quorum, a conservative's abstention was passive. He or she would desert the ballots because he or she had no incentive to vote (each individual vote had no impact on the final outcome). With a participation quorum, conservatives' abstention was strategic. They abstained because of a disincentive to vote (each individual conservative vote would contribute to an undesirable outcome, by helping the quorum to be reached). These implications are at odds with Herrera and Mattozzi's equivalence result between the different types of quorum requirements.

We also analyzed the effects of increasing the percentage of consumption voters to 35% (not shown). As expected, some of the equilibria disappeared. In the case of a participation quorum, we no longer have massive abstention from conservatives (equilibria 3 and 4 of Table 7 are no longer equilibria). However, the no show equilibrium of Table 1 still survives (although in this case it corresponds to a turnout of 35%). In the case of the approval quorum, equilibrium 4 and 5 of Tables 1 and 7 disappear, meaning that supporters always achieve the quorum. This result is also to be expected. Given that at least 35% of changers vote, no matter what (corresponding to 17.5% of the electorate) reaching the quorum (of 25%) is now easy to accomplish. The other equilibria survived and they are very similar to the ones described in Table 7 (with higher participation rates, naturally.

Finally, we have also considered the case of $\bar{c} = 1$ (not shown), implying that 50% of the electorate faces negative voting costs. In this case, only equilibrium 1 survived with an approval quorum. With the participation quorum two equilibria survived. Equilibrium 1 is similar to the previous cases. In equilibrium 2, some of the consumption voters decide to abstain, in order to avoid meeting the quorum.[28]

From these simulations, one concludes that if the number of consumption voters is sufficiently (but rather plausibly) low (25% in our simulations), nothing essential changes in our analysis. Then, as we increase this percentage, some of the multiple equilibria start to disappear. The first equilibria to disappear are the ones in which the quorum is not satisfied, implying that the equilibria that survive are the ones biased towards 'Change'. Of course, if the percentage of consumption voters is high enough, quorum requirements are no longer binding and the quorum equilibria are just equal to the no quorum equilibrium

7. Conclusions

The institutional design of referenda is crucial for ensuring correct representation of collective preferences. The introduction of quorum requirements in referenda has typically been justified as a means of preventing 'distortions' in voting outcomes due to low turnout: by allegedly preventing adoption of policy changes supported only by a minority of the population, quorum restrictions were supposed to provide greater legitimacy to voting outcomes. However, we have shown that quorum requirements can have detrimental consequences, including introducing some of the very distortions that they are ostensibly intended to prevent. We have provided several examples that suggest the presence of these consequences in real world situations.

We used a standard pivotal voter model and computed equilibria for different scenarios. Our analysis has shown that, although turnout and approval quorum rules do not have exactly the same consequences, they are similar insofar as they can fail to achieve their purported goals. First, because they may promote abstention, quorum requirements contradict the aim of lending legitimacy to the election results. Second, rather than

[28] This second equilibrium also disappears if one assumes that the percentage of the electorate that has negative voting costs is higher than 55%.

biasing the results in favor of the status quo, the bias may be on the contrary against the status quo. Third, it is not clear that quorum rules disadvantage small decisive minorities; actually, we have shown that they may facilitate such minorities achieving their objectives. Finally, quorum requirements may create an additional problem: we have described several quorum equilibrium outcomes in which a whole group of voters, typically conservative, does not vote. In turn, this implies that voting is no longer secret. This introduces possibilities of undemocratic forms of pressure, of which there is clear historical and also more contemporary evidence.

In sum, we have shown that distortions caused by quorum requirements are such that, when considering the results of a particular referendum, it becomes extremely difficult to make unambiguous inferences about the true preferences of the electorate.

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